Robust time-domain source stepping for DC-solution of circuit equations

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Summary. Most analyses of circuit equations start with solving the steady-state (DC) solution. In several cases this can be very hard. We present a novel time domain source stepping procedure to obtain a DC solution of circuit equations. The source stepping procedure is automatically adaptive. Controlled sources can be elegantly dealt with. The method can easily be combined with existing pseudo-transient procedures. The method is robust and efficient.

1 Introduction

The circuit equations can be written as [5, 10]

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{q}(\mathbf{x}) + \mathbf{j}(\mathbf{x}) + \mathbf{s}(t, \mathbf{x}) = 0 \qquad (1)$$

Here $\mathbf{s}(t, \mathbf{x})$ represents the specifications of the sources. The unknown $\mathbf{x} = \mathbf{x}(t)$ consists of nodal voltages and of currents through voltage defined elements. We assume that $\mathbf{q}(0) = 0$, and $\mathbf{j}(0) = 0$.
The steady state solution, which is called DC-solution (Direct Current solution), $\mathbf{x}_{\mathrm{DC}}$, satisfies

$$\mathbf{j}(\mathbf{x}_{\mathrm{DC}}) + \mathbf{s}(0, \mathbf{x}_{\mathrm{DC}}) = 0. \qquad (2)$$

Usually, and already hinted by setting $t = 0$ in (2), the DC-solution provides the initial value for the transient problem (1). In general, the problem (2) is non-linear. How to solve this problem is the subject of this note.
The importance of the DC-problem lies in the fact that the DC-solution is crucial as starting solution for a number of next analyses (transient analysis, AC analysis, Harmonic Balance analysis, Periodic Steady-State analysis). In general, (1) forms a system of Differential-Algebraic Equations (DAEs). With $\mathscr{C} = \left.\frac{\partial \mathbf{q}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{\mathrm{DC}}}$ and $\mathscr{G} = \left.\frac{\partial \mathbf{j}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{\mathrm{DC}}}$. We assume that $\lambda\mathscr{C} + \mathscr{G}$ is non-singular for λ in some neighbourhood of 0 (may be excluding $\lambda = 0$). To solve the equations Newton's method, or variants, may be applied [3,5,8], which can be combined with $g_{\min}$-stepping, in which linear conductors g are placed parallel to the non-linear part inside each transistor (device). Iteratively $g \downarrow g_{\min}$, after which the Newton counter is increased.
Another approach is Pseudo-Transient [2]. In Pseudo-Transient (PT) one can use relaxed tolerances for the Newton process and for the time step control procedure. Also this can be combined with $g_{\min}$-stepping during each time step. In PT one has to provide a non-trivial initial solution. A new procedure is decribed in the next section. Other methods are: temperature stepping, source stepping (the sources are iteratively increased to their final value), homotopy methods, or optimization [1, 4, 7, 9–12].

2 Time-domain Source Stepping

Usually, in Source Stepping one introduces a parameter λ and considers the problem

$$\mathbf{j}(\mathbf{x}(\lambda)) + \lambda\mathbf{s}(0, \mathbf{x}(\lambda)) = 0. \qquad (3)$$

In this case it is assumed that for $\lambda = 0$ the problem (3) is easily solved so that in the end the original problem is solved. The same parameter λ is applied to all sources s in the circuit. In general, for each value of λ a nonlinear problem has to be solved.
We introduce a time-domain variant (SSPT) that offers an automatic continuation process, based on PT and adapting the transient stepsize and the λ stepsize at the same time.
We define a time $t = T$ at which we want to have solved the original DC-problem. We also introduce a time $T_\alpha = \alpha T$ (by default $\alpha = 0.5$) at which ordinary PT will start simulation using the sources as in the original DC-problem, i.e. using $\lambda = 1$ and where PT integrates from T_α to T', where $T' \leq T$ is the point where all transient effects have become negligible (see also Fig. 1).
On the interval $[0, T_\alpha]$, a special PT integration is performed with the function $\lambda(t) = t/T_\alpha$. Hence, at each time step, also the actual applied source values change. The interval $[0, T_\alpha]$ is the switch-on interval, the interval $[T_\alpha, T]$ is the interval to damp-out transient effects. On both intervals PT uses an automatic time step determination procedure. On the interval $[T_\alpha, T]$ an ordinary PT procedure is executed. Hence,

if, at some time point, the Newton iterative process does not converge, a re-integration will be done with a smaller stepsize. Recursion in controlled sources asks

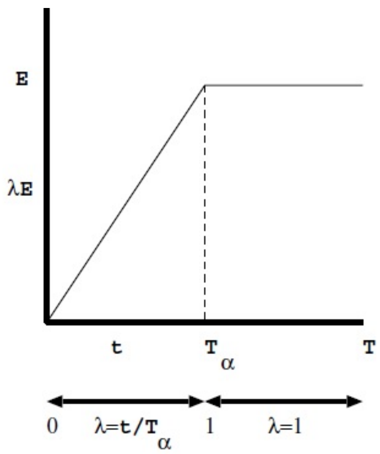

Fig. 1. On $[0, T_\alpha]$ a time-dependent voltage source $\lambda(t)\mathbf{E}$ is used where $\lambda(t) = t/T_\alpha$. On $[T_\alpha, T]$ we have $\lambda \equiv 1$.

for a modification in (3). An expression for a controlled voltage source $E_1(0,1)$ may look like

$$V(E_1) = 5 + 4I(E_1) + [6V(R_1) + 7I(E_2) + 12]^2 \quad (4)$$

It is controlled by the controlling "ev's" (electrical variables) $I(E_1)$, $V(R_1)$, and $I(E_2)$. We write the expression for the applied value $V(E_1)$ as

$$V(E_1) = \psi(\mathrm{ev}_1, \mathrm{ev}_2, \ldots, \mathrm{ev}_n) \quad (5)$$

As value during the source stepping at time t on $[0, T_\alpha]$ we propose to take

$$V(E_1) = \tilde{\psi}(\mathrm{ev}_1, \ldots, \mathrm{ev}_n), \text{ where} \quad (6)$$

$$\tilde{\psi}(\mathrm{ev}_1, \ldots, \mathrm{ev}_n) = \psi(\mathrm{ev}_1, \ldots, \mathrm{ev}_n) + (\lambda(t) - 1)\psi(0, \ldots, 0). \quad (7)$$

Note that in (4), $\psi(0, \ldots, 0) = 149$. This value has to be calculated once. When in (4) E_2 is a controlled voltage source too, contributions to the Jacobian matrix are calculated by $\frac{\partial \tilde{\psi}}{\partial \mathbf{x}} = \frac{\partial \tilde{\psi}}{\partial \mathrm{ev}_i} \frac{\partial \mathrm{ev}_i}{\partial \mathbf{x}}$, which gives recursion. Note that λ does not occur in the matrix. Clearly, for $\lambda = 0$ the applied voltage is zero (assuming starting from the zero solution, which implies that all ev's are zero), which makes the zero solution the exact solution.When $\lambda = 1$ the original voltage expression is used. Since our equations (1) are DAEs we remark that for all t the generated solution is consistent for the problem at hand. Because of the switch-on and the damp-out phase the process mimics a real physical process.

3 Results

We tested the SSPT on a set of difficult problems where parameters were swept (temperature, and statistics). The SSPT was always convergent (without needing $g_{\min}$-iteration). It was 1-13 times faster than Newton-Raphson (that sometimes needed internal $g_{\min}$-iteration). Normal PT was less robust than SSPT.
Further improvements in the time-domain integrations, after starting with a proper $\mathbf{x}_{\mathrm{DC}}$, have been tuned to fault analysis [6].

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