Uncertainty Quantification of Inrush Currents in Electric Machines with Respect to Measured Material Data

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Summary. The startup of most electrical machines exhibits a strong nonlinear behavior due to saturation. In practice, the underlying nonlinear saturation curve is modeled according to measurement data that typically contain errors. The electromagnetic fields and in particular the inrush currents inherit this uncertainty. In this paper, we propose a specific stochastic model (BH-curve) to describe uncertainties and we demonstrate the use of generalized polynomial chaos for the uncertainty quantification of these inrush currents. This requires time stepping of systems of nonlinear partial differential algebraic equations that result from the coupling of field and circuit systems.

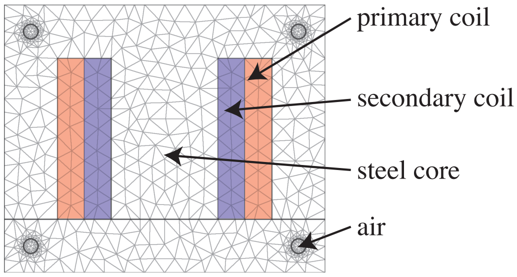

Fig. 1: 2D model of a transformer, taken from [9].

1 Introduction

Efficient design of electric machines (transformers, actuators, generators etc.) requires insight into the device's electromagnetic field distribution. Often, the available inputs, e.g. material data, include unknown errors for example due to measurements. The influence of these errors can be characterized by uncertainty quantification. In the mathematical models, the corresponding parameters are substituted by random variables to describe the uncertainties.

In this paper a transformer, modeled by the magnetoquasistatic approximation to Maxwell's partial differential equations (PDEs), is considered. This system is coupled to a network model of an electric circuit given by a system of differential algebraic equations (DAEs). The coupling is necessary in order to to simulate the startup phase where the highest inrush currents can be observed. To account for the measurement errors, the material curves include (random) parameters, such that the time-dependent solution of the PDAEs becomes a random process.

Uncertainties in the material parameters of magnetoquasistatic problems have been studied before but only considering linear material laws in frequency domain, e.g., [2, 3, 11]. In this paper we do not propose to model the material laws as uncertain, but the underlying measurement data. This allows for a natural choice of the probability distribution.

The stochastic model can be solved by a quasi Monte-Carlo simulation, for example. We use the generalized polynomial chaos (gPC), see [1, 6, 12], in the numerical simulation to investigate how this approach behaves. A stochastic Galerkin method results in a larger coupled system of DAEs, cf. [10]. To illustrate the modeling and the simulation, we discuss a 2D finite element discretization of a transformer.

2 Field Model

In the low-frequency regime the electromagnetic field, i.e., the eddy current problem, is typically described in terms of the magnetic vector potential $\mathbf{A}$ (MVP), with magnetic flux density $\mathbf{B} = \nabla \times \mathbf{A}$, on a computational domain by the curl-curl equation

$$\kappa \frac{\partial \mathbf{A}}{\partial t} + \nabla \times \left(\nu \nabla \times \mathbf{A} \right) = \mathbf{J}_{\mathrm{src}} \qquad (1)$$

with conductivity κ and nonlinear reluctivity ν. In our model, $\nu = \nu(\nabla \times \mathbf{A}, \mathbf{Y})$ may depend on random variables $\mathbf{Y}$ to account for measurement errors. The system is equipped with boundary and initial conditions for $\mathbf{A}$. The material parameters are piecewise constant in all subdomains, only for ferromagnetic materials (e.g. the steel core in Fig. 1) the Brauer model, [4], is chosen to account for nonlinear saturation

$$\nu(\mathbf{B}, \mathbf{Y}) = k_1(\mathbf{Y}) \cdot \exp\left(k_2(\mathbf{Y}) \cdot |\mathbf{B}|^2\right) + k_3(\mathbf{Y}) \,, \qquad (2)$$

where the model parameters k_i are fitted from measurement data and thus depend on the errors described by $\mathbf{Y}$.

The circuit coupling is established by identifying parts of the computational domain as branches in the circuit. Typically for coils the stranded conductor model is used and for massive bars the solid conductor model is feasible, [5]. In the case of a number of N_{str} stranded conductor models (i.e., spatially resolved field elements), the excitation from the circuit is imposed on the field by the source term

$$\mathbf{J}_{\text{src}} = \sum_{k=1}^{N_{\text{str}}} \chi_k i_k \,, \tag{3}$$

where the winding functions χ spatially distributes the corresponding currents $\mathbf{i}$. To obtain current/voltage relations for each field element, additional coupling equations are needed, e.g.

$$\int_{\Omega} \chi_k \cdot \frac{\partial \mathbf{A}}{\partial t} \, \mathrm{d}V = v_k - R_k i_k \quad (k = 1, \ldots, N_{\text{str}}) \tag{4}$$

with the DC resistances $\mathbf{R}$ for the windings. Hence, given voltage drops $\mathbf{v}$, the system (1-4) defines $\mathbf{A}, \mathbf{i}$.

3 Uncertainties in the Measurement

The material parameter ν is implicitly given by measurements of the BH-curve (B_i, H_i), for $i = 1, \ldots, N$. The Brauer material model (2) can be fitted either by a nonlinear least squares algorithm, as e.g. in [8], or less elegantly by selecting 3 measurement points and computing the reluctivity function that fulfills

$$H_i = \nu(B_i) B_i$$

exactly where we choose the points $i = 1, 2, 3$ without loss of generality, e.g. [7]. We follow the second approach to keep the parameter space small.

The field strength H is assumed to be affected by a measurement error:

$$\Big(B_i, H_i \cdot Y_i\Big) \qquad \text{for } i = 1, 2, 3$$

where Y_i is normally distributed with mean $\mu = 1$ and standard deviation $\sigma = 0.1$.

We propose to quantify the impact of the perturbations above on the currents $\mathbf{i}$ in (4) by the generalized polynomial chaos.

The transformer model as depicted in Fig. 1 has been simulated for 100 realization of the above introduced normally distributed random variables. The results are shown in Fig.2. The uncertainties cause deviations of up to 20A in the primary inrush current.

In the full paper the computation of the expected values and the variance of the currents are discussed in more detail and using different uncertainty quantification techniques.

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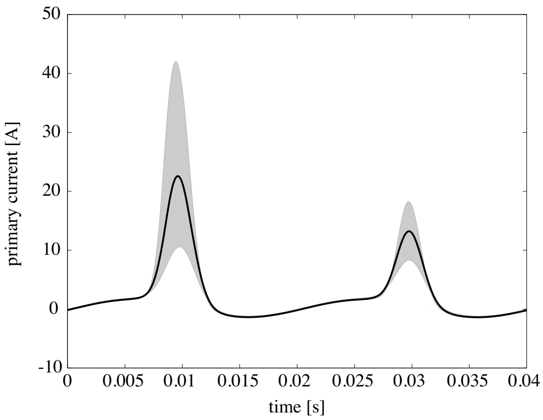

Fig. 2: Impact of random parameters on the primary current. The black line denotes the mean of the 100 random walks, the grey neighborhood is given by the maxima and minima.

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