

# Computation of Optimal Model Parameters of an Extended Brauer Model for Ferromagnetic Material Behaviour

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**Summary.** Simulation of low-frequency magnetic fields in electric machines demands for implicit time integration. For the nonlinear reluctivity of the ferromagnetic yoke, a smooth material curve is needed to avoid convergence problems in Newton's method. In this paper the Brauer model is extended to fit the material behavior at low fields more accurately and to guarantee physically correctness for high fields. Furthermore, a procedure to obtain optimal parameters is developed and discussed using a numerical example.

## 1 Introduction

Typically Finite Element (FE) based simulations of eddy currents use the magnetic vector potential  $\mathbf{A}$  and the curl-curl equation

$$\sigma \frac{d\mathbf{A}}{dt} + \nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}_s, \quad (1)$$

with conductivity  $\sigma$ , reluctivity  $\nu$  and source current density  $\mathbf{J}_s$ . For iron parts the material relation  $\mathbf{H}(\mathbf{B}) = \nu \mathbf{B}$  becomes nonlinear, where  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic flux density and  $\mathbf{H}$  the magnetic field strength. We neglect anisotropy and hysteresis. Hence, we can apply  $H = \nu B$  in terms of  $H := \|\mathbf{H}\|_2$  and  $B := \|\mathbf{B}\|_2$ . Typical models are spline interpolations of measurements, [3], and Brauer's model, [1]:

$$H_{br}(B) = \nu_{br}(B^2)B = (k_1 e^{k_2 B^2} + k_3)B.$$

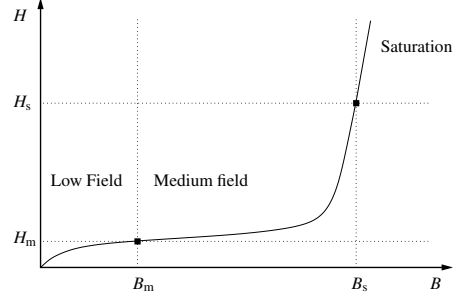
Both allow a simple calculation of the reluctivity  $\nu = \nu(B^2)$  and its derivative  $\frac{d}{dB^2} \nu$ , needed in the computation of material matrices occurring in the space discretization of (1), see e.g. [2]. Brauer's model is well understood, e.g., a sensitivity analysis shows that currents and fluxes through machines are most sensitive w.r.t. to perturbations in  $k_2$  followed by  $k_1$  and  $k_3$ . The model is sufficiently accurate for medium fields but the behavior for low fields (Rayleigh region) and high fields (full saturation, i.e.,  $\frac{dH}{dB} = \nu_0$  as for vacuum) cannot be represented accurately.

## 2 Extended Brauer model

For high fields the material behaves like vacuum

$$H_{sat}(B) = \nu_0(B - B_s) + H_s.$$

For low fields the dependence of  $B$  on  $H$  is quadratic:



**Fig. 1.** Regions of the  $H(B)$ -curve.

$$H_{ray}(B) = \sqrt{\frac{1}{4\alpha^2 \nu_{init}^2} + \frac{B}{\alpha}} - \frac{1}{2\alpha \nu_{init}}$$

with initial reluctivity  $\nu_{init}$  and Rayleigh constant  $\alpha$ . In combination we obtain a  $H(B)$ -curve as shown in Fig. 1, with interface points  $(B_m, H_m)$  and  $(B_s, H_s)$ .

Let us determine the coefficients of an extended Brauer model s.t. the global model is continuous differentiable. The classical model does not simply allow to replace low field and high field parts. A shift of the Brauer model enables to fulfill the continuity and differentiability conditions at  $B_m$ :

$$H_{ebr}(B) = (k_1 (e^{k_2(B-B_m)^2} - 1) + \nu_{d,m})(B - B_m) + H_m$$

with  $\nu_{d,m} := \frac{dH}{dB} H_{ray}(B_m)$  the differential reluctivity at the end of the Rayleigh region. We define the function

$$H(B) := \begin{cases} H_{ray}(B) & \text{if } 0 \leq B < B_m, \\ H_{ebr}(B) & \text{if } B_m \leq B < B_s, \\ H_{sat}(B) & \text{if } B_s \leq B. \end{cases} \quad (2)$$

To fix the model parameters  $k_1$  and  $k_2$ , we use the continuity conditions  $H_{ebr}(B_s) = H_s$  and  $\frac{dH}{dB} H_{ebr}(B_s) = \nu_0$ , and solve each equation for  $k_1$ :

$$k_1 = \frac{\frac{H_s - H_m}{B_s - B_m} - \nu_{d,m}}{e^{k_2(B_s - B_m)^2} - 1}, \quad (3)$$

$$k_1 = \frac{\nu_0 - \nu_{d,m}}{(2k_2(B_s - B_m)^2 + 1) e^{k_2(B_s - B_m)^2} - 1}. \quad (4)$$

From (3) and (4) we find a nonlinear equation for  $k_2 > 0$ , which is solvable under the conditions  $\nu_0 > \nu_{d,m}$

$$0 < \frac{\frac{H_s - H_m}{B_s - B_m} - \nu_{d,m}}{\nu_0 - \nu_{d,m}} < \frac{1}{3}$$

and  $B_s > B_m > 0$ ,  $H_s > H_m > 0$ ,  $\nu_0 > \nu_{d,m}$ .

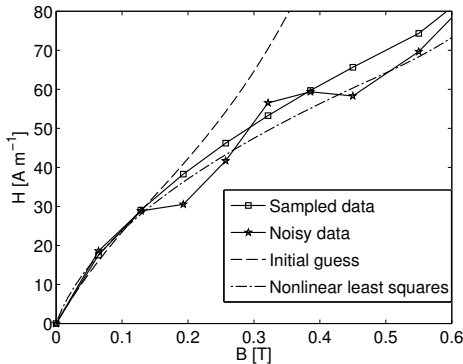


Fig. 2. Low field region.

The Rayleigh constant can be obtained by  $\alpha := (B_m/H_m - 1/v_{\text{init}})/H_m$ . Therefore the model (2) can be fixed from the data  $\mathbf{p} := (v_{\text{init}}, B_m, H_m, B_s, H_s)$ .

### 3 Optimal Model Parameters

Let us consider measurement points  $(B_j, H_j)$ ,  $j = 1, \dots, N$ , of a ferromagnetic material. We assume monotonicity of the data and a distinctive Rayleigh region.

To find optimal model parameters we solve a nonlinear least squares (NLS) problem, i.e.,

$$\min_{\mathbf{p}} \sum_{j=1}^N \frac{(H(B_j; \mathbf{p}) - H_j)^2}{H_j^2}. \quad (5)$$

Proper initial guesses are important because of the nonliterary. A procedure for this purpose is given by:

- i) The fraction of first non-zero measurement points approximates  $v_{\text{init}}$ . Then find index  $r$  that minimizes  $(H_{r+1} - H_r)/(B_{r+1} - B_r)$ .  $(B_r, H_r)$ . It is an approximation to  $(B_m, H_m)$ . Compute  $\alpha$  and  $v_{d,m}$ .
- ii) Approximate the beginning of the saturation region with the last measurement points. Solve (3) and (4) for  $k_1$  and  $k_2$  by replacing  $v_0$  with the secant slope  $v_N$  of the last two measurement points.
- iii) If  $v_N \approx v_0$ , use the last point as approximation of  $(B_s, H_s)$ . Otherwise solve  $\frac{dH}{dB} H_{\text{ebr}}(B_s) = v_0$  for  $B_s$  and compute  $H_s = H_{\text{ebr}}(B_s)$ .
- iv) Solve the NLS problem (5).

### 4 Example

For validation we created test data from (2) using  $v_{\text{init}} = 400 \text{ mH}^{-1}$ ,  $(B_m, H_m) = (0.5 \text{ T}, 70 \text{ A m}^{-1})$  and  $(B_s, H_s) = (2 \text{ T}, 100 \text{ kA m}^{-1})$ . We sample the first two regions with 8 equidistant points. We incorporate Gaussian measurement noise by  $\tilde{H}_i = \max(0, H_i \cdot (1 + \sigma X_i))$ , where  $X_i \sim \mathcal{N}(0, 1)$  and  $\sigma = 0.1$ . Fig. 2 and 3 show the fitted results. The proposed initial guess is sufficient to achieve convergence of the NLS problem. We obtain a curve close to the original curve.

The extended Brauer model is tested in a 2-D FE simulation of a transformer at no-load. Simulation results with the extended Brauer model with parameters

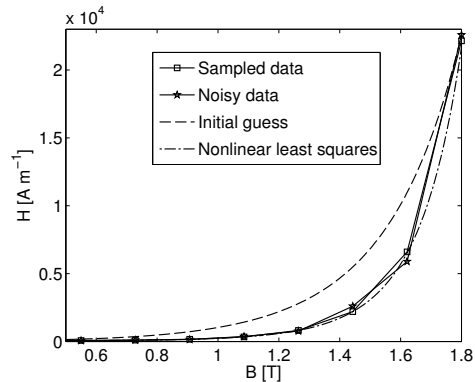


Fig. 3. Medium field region.

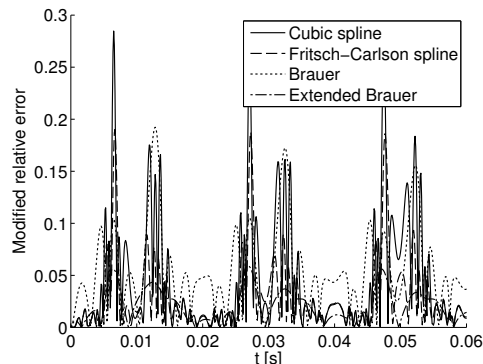


Fig. 4. Errors  $|i - \hat{i}|/(|i| + 0.1)$  of no-load current. The current peaks are at 0.01, 0.03 and 0.05.

from above are used as a reference. For the spline interpolation (cubic and Fritsch-Carlson spline) we smoothed the noisy samples in the Rayleigh region with a moving average filter. Errors of the no-load current  $i$  through the device are depicted in Fig. 4.

The original Brauer model is the computationally least expensive but yields large errors in comparison to the reference solution. It cannot match the shape of the material curve for low and high fields. Spline interpolated measurements yield medium errors but need a high number of Newton steps to converge, cf. [4]. The extended Brauer model is only slightly more expensive than the original Brauer model but is the most accurate also in Fig. 4. It also recovers the reference curve.

### References

1. J. R. Brauer. Simple Equations for the Magnetization and Reluctivity Curves of Steel. *IEEE Trans. Magn.*, 11(1):81, 1975.
2. H. De Gerssem, I. Munteanu, and T. Weiland. Construction of Differential Material Matrices for the Orthogonal Finite-Integration Technique With Nonlinear Materials. *IEEE Trans. Magn.*, 44(6):710–713, 2008.
3. B. Heise. Analysis of a fully discrete finite element method for a nonlinear magnetic field problem. *SIAM J. Num. Ana.*, 31(3):745–759, 1994.
4. H. Vande Sande, F. Henrotte, and K. Hameyer. The Newton-Raphson method for solving non-linear and anisotropic time-harmonic problems. *COMPEL*, 23(4):950–958, 2004.