Broad Band Surface Impedance Boundary Conditions for Higher Order Time Domain Discontinous Galerkin Method

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Summary. An implementation of the broad band Surface Impedance Boundary Condition (SIBC) for the high order Discontinuous Galerkin (DG) method in the time domain is presented. In order to treat the frequency dependent impedance function a set of auxiliary differential equations is introduced. The effect of the DG approximation order on the accuracy will be studied, and the results will be compared with the conventional time domain Finite Element Method.

1 Introduction

Time domain modeling is very attractive for wide band electromagnetic problems, since it allows to compute for a large range of frequencies in a single simulation. However, when the frequency band of interest is wide, the dispersive nature of material parameters, i.e. their variation with respect to frequency, needs to be considered. In order to model dispersive electromagnetic materials in time domain simulations, one generally needs to evaluate one or more convolution integrals. Clearly a direct computation of convolution terms is too expensive for every practical computation. For this purpose, several numerically efficient approaches have been proposed. One approach is a recursive convolution [7]. Another technique which is particularly suited for explicit time domain simulations is the Auxiliary Differential Equation (ADE) method. In the following, ADE is applied in the context of SIBC for arbitrary frequency dependent electric conductivities. Finite Difference Time Domain method (FDTD) [11] is widely used for time domain simulations. It leads to explicit time stepping and it is straightforward to implement. However, FDTD has a two important disadvantages: First, the method loses substantial accuracy at curved geometrical boundaries. Second, FDTD is at most 2nd order accurate, thus, it suffers under large numerical dispersion errors at high frequencies. Finite Element Method (FEM) [12] is very accurate as far as the modeling of arbitrary geometries is concerned. However, the time domain FEM leads to implicit time stepping [5], and is therefore numerically extremely expensive. The Time Domain Discontinuos Galerkin Method (DG) [3] combines the advantages of the aforementioned methods: it is free of numerical dispersion, modeling of arbitrary geometries is straightforward, and due to the global discontinuity of the basis functions, the resulting time stepping scheme is explicit. However, due to the discontinuity of basis functions at cell interfaces, unphysical spurious modes will occur. A possible cure to the problem of spurious modes is the application of various penalization methods as proposed, e.g., in [3], [1].
In this study, we will describe the implementation of a wide band SIBC for higher order DG by means of the ADE method. Furthermore, the effect of discretization order, rational approximation order for the impedance function as well as the impact of penalization on the accuracy of DG simulations with SIBC will be investigated.

2 DG Method

In this study, we will consider the Maxwellian initial value problem. The three-dimensional computational domain Ω is discretized into N non-overlapping elements, and on the boundary $\partial\Omega$, the SIBC is applied. Within an element, the electric field **E** and the magnetic flux density **B** are approximated by a linear combination of vectorial basis functions ϕ_E and ϕ_B, respectively. As both of the basis functions, ϕ_E and ϕ_B, are defined cell-wise without global continuity, in the DG method, a numerical flux approach is applied in order to impose the neccessary continuity at the interfaces between mesh cells in the weak sense. A detailed despription of the method as well as of the approximation functions, ϕ_E and ϕ_B, used in the present implementation is given in [1].

3 The SIBC Approach

Modeling of media with large but finite electrical conductivities typically leads to very dense meshes and thus to small time steps as required for stability in explicit time domain simulations. Therefore, it is desirable to exclude the lossy media from the computational domain. This can be done by introducing at the boundary surface of the conductive domain impedance-like conditions, which provide a relationship between the tangential electric field to the tangential magnetic field components. The classical SIBC was introduced by Leontovich (cf. [10]). It assumes the lossy surface to be planar and ignores the tangential variation of the field quantities. The error of the Leontovich SIBC is order of $O(\delta^2)$, where δ is skin depth, which makes it especially suitable for high frequencies. [4]. The second order SIBC [6] takes into account also the curvature of the surface. It is, furthermore, possible to construct higher order, thus, more accurate SIBC by taking into account, in addition, the tangential variation of the field components along the lossy surface [8]. When the thickness of the conductive medium is of the order of skin depth, the electromagnetic fields on the different sides of lossy medium interact with each other. Also this type of problems can be modeled by means of SIBC, using e.g. Sarto's [9] approach.

4 Approximation of Impedance Function

In order to transform the dispersive impedance function into the time domain, it is first approximated in the frequency domain as a series of rational functions [2]. The rational approximation for the tangential magnetic field can be written as:

$$Y(\omega)\mathbf{E}_t \approx Y_0\mathbf{E}_t + \sum_{i=1}^{P} \frac{Y_i\mathbf{E}_t}{j\omega - \omega_i}, \qquad (1)$$

where $\mathbf{E}_t$ is tangential electric field on the surface, P is the order of the rational approximation, Y_0 free space admittance, Y_i and ω_i are approximation parameters. Let us rewrite the rational approximation given in (1) as $Y(\omega)\mathbf{E}_t \approx \mathbf{Y}_0 + \sum_{i=1}^{P}\mathbf{Y}_i$. The the SIBC condition transforms in the time domain to

$$\mathbf{Y}_0 = Y_0\mathbf{E}_t \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{Y}_i - \omega_i\mathbf{Y}_i = Y_i\mathbf{E}_t. \qquad (2)$$

Equation (2) represent the auxiliary differential equations of the ADE method which need to be solved for in the time domain together with the full set of Maxwell's equations.

5 System of Equations

The system of discrete equations to be solved in the time domain can be written as:

$$\begin{cases} \mathbf{C}_\mathrm{E}\mathbf{e} + \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}_\mu\mathbf{h} = 0 \\ \mathbf{C}_\mathrm{H}\mathbf{h} - \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}_\varepsilon\mathbf{e} = \mathbf{C}_\mathrm{Y}\sum_{i=0}^{P}\mathbf{Y}_i \\ \mathbf{Y}_0 = Y_0\mathbf{e}_t \\ \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{Y}_i - \omega_i\mathbf{Y}_i = Y_i\mathbf{e} \quad \text{for } i = 1 \ldots P, \end{cases} \qquad (3)$$

where $\mathbf{C}_\mathrm{E}$ and $\mathbf{C}_\mathrm{B}$ are curl-matrices obtained by high order DG discretization, $\mathbf{C}_\mathrm{Y}$ is so called "admittance flux" matrix, and $\mathbf{M}_\mu$ and $\mathbf{M}_\varepsilon$ are block-diagonal mass matrices. In the full paper, the numerical accuracy and efficiency of this approach with respect to discretization order for different rational function approximations (1) will be discussed.

6 Summary

Dispersive SIBC will be implemented for time domain DG method in order to model a wide frequency band at a single simulation. The frequency dependent conductivity of lossy surfaces is considered in time domain by auxiliary differential equations. We will study the accuracy of the solution for different DG discretization orders and impedance function approximations, and compare our results with the standard SIBC-FDTD method.

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