Frequency Parameterized Models for Planar On-Chip Inductors

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Summary. The paper proposes an efficient method for the modeling of high frequency electromagnetic field effects, such as skin or proximity effects, inside on-chip metallic conductors. Compact sub-models obtained by using an electromagnetic field discretization approach based on the Finite Integration Technique in which frequency dependent Hodge operators are used, are connected to magnetic circuits that describe inductive couplings.

1 Frequency Dependence

Designers of integrated circuits require models of passive components which describe all relevant electromagnetic field effects at high frequency. These effects are quantified by the Maxwell equations of the electromagnetic field. In the Finite Integration Technique (FIT), by applying the global form of electromagnetic field equations on the mesh elements (elementary faces and their borders), a system of differential algebraic equations, called Maxwell Grid Equations (MGE) is obtained [2]. Due to high conductivity ($\sigma \gg \omega\varepsilon$), the electromagnetic field inside metallic conductors can be considered a magneto-quasi-static (MQS) one. MGE for MQS regime are combined with the Hodges operators, which describe the material behavior

$$\begin{aligned} &\mathbf{B} = \mu\mathbf{H}, && \mathbf{J} = \sigma\mathbf{E} \quad \Rightarrow \\ \Rightarrow \quad &\varphi = \mathbf{M}_\mu \mathbf{u}_m = \mathbf{M}_\nu^{-1}\mathbf{u}_m, && \mathbf{i} = \mathbf{M}_\sigma \mathbf{u}_e, \end{aligned} \qquad (1)$$

where the following global variables have been used: electric and magnetic voltages $\mathbf{u}_e$, $\mathbf{u}_m$, and magnetic fluxes φ and conduction currents $\mathbf{i}$, that are associated to the grids elements in a coherent manner.

In the classical FIT approach, the discrete Hodge operators $\mathbf{M}_\nu$ and $\mathbf{M}_\sigma$ and are constant diagonal matrices, which can be built by independent averaging of material constants $\nu = 1/\mu$ and σ over each cell. In order to describe field effects at high frequency such as skin and proximity effects, the cell dimensions have to be much less than the skin depth $\delta = \sqrt{2/(\omega\mu\sigma)}$, which is 6.7$\mu$m for Cu at 100 GHz and 15 μm at 20 GHz. In order to keep the number of cells at a reasonable level, non-uniform grids could be used, with peripheral cells smaller than internal ones. Even so, the number of cells required by a reasonable accuracy can be relatively high. To avoid this drawback, it was proposed to replace the Hodge operators used in classical FIT with others appropriate for the description of high field effects in conductors [3].

By solving the complex Helmholtz equation for the electric field in a rectangular homogeneous cell having the conductivity σ and the permeability μ, of dimensions: a (along the Ox axis), $2b$ and $2c$ (along Oy and Oz, respectively), we found that the complex admittance of the cell along the Ox direction is

$$\mathbf{Y} = \frac{8}{\pi^2 R_0} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \left[\frac{\tanh(\lambda_k b)}{\lambda_k b} + \frac{\tanh(\mu_k c)}{\mu_k c} \right], \qquad (2)$$

where $R_0 = a/(4\sigma bc)$ is the D.C. resistance of the analyzed cell along the Ox direction, and the complex numbers λ_k and μ_k are given by

$$\lambda_k = \sqrt{\gamma^2 + \left[\frac{(2k-1)\pi}{2c}\right]^2}, \qquad (3)$$

$$\mu_k = \sqrt{\gamma^2 + \left[\frac{(2k-1)\pi}{2b}\right]^2}, \qquad (4)$$

where $\gamma^2 = i\omega\mu\sigma$ is the complex diffusion constant in the conductor. Relation (2) does a smooth connection between the D.C. value R_0 and the value given by a strong skin depth formula $a/(4(b+c)\delta)$ (Fig. 1).

The first results carried out in FIT with FredHO on a simple test case having an analytical solution taken as reference, are given in Table 1 and Fig. 1 and show its efficiency both with respect to the computational effort and error. FredHO is able to catch not only the dependence of the A.C. rezistence with respect to the frequency, but also the frequency dependence of the conductor inner inductivity.

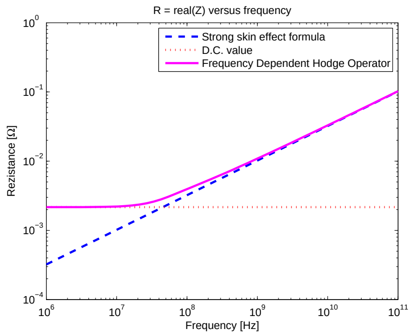

Fig. 1. Dependence of the A.C rezistence $R_{AC} = \mathrm{real}(1/Y)$ with respect to the frequency.

Table 1. Validation of FIT with FredHO

	No. DoFs.	Relative error [%]
Analytic	-	0
Classical FIT, uniform grid	770	31
Classical FIT, non-uniform grid	667	3
FIT with FredHO	5	0.0006

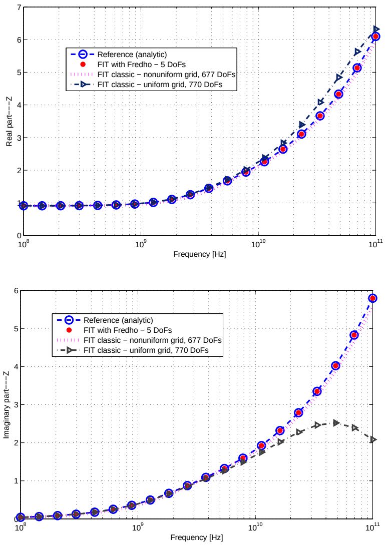

Fig. 2. Frequency characteristic - with or without FredHO.

2 Inductive Effects

Our models of the inductors consist of two coupled circuits: an electric and a magnetic one. The electric circuit includes frequency dependent reactances inside conductors and RC sub-circuits outside, whereas the the magnetic circuit describes inductive (magnetic) couplings. Magnetic hooks are placed in the holes of each fundamental loop of the electric circuit.

The magnetic circuit is driven by ”voltage” sources (actually magnetic voltage sources), controlled by the independent loop currents of the electric circuit (co-tree currents). The electric induced voltages are modeled by voltage sources placed in the co-tree branches of the electric circuit, which are controlled by the time derivative of the magnetic circuit ”currents” (actually magnetic fluxes). The magnetic reluctances are associated to fundamental current loops and they are extracted directly from the field solution. The controlling by time derivatives of magnetic fluxes is obtained by means of a third ”derivative” subcircuit. Thus, the model extracted becomes compatible with any circuit simulator, including standard Spice. Contrary to VPEC or other partial-inductance/reluctance approaches [5, 6] our reluctances are associated to fundamental current loops and they are extracted directly from the field solution. By using the loop-reluctance matrix instead of partial reluctances/inductances (”K elements”), the sparsification is very effective and robust (the passivity is not lost), which is an essential request [1, 4].

3 Conclusion

In frequency domain simulation, high frequency field effects can be taken into consideration in a very effective manner, if the Hodge operators depend on the frequency. From the computational resources point of view, this is more efficient than using a fine discretization grid inside the conductors, even if this implies some matrix re-assembling at every frequency sample. In this paper this technique is combined with the use of magnetic circuits describing inductive effects in order to obtain compact models for planar inductors. Our presentation will describe in detail this technique and will show results for real benchmarks.

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